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On the effects of (partial) quenching on penguin contributions to $K \to \pi\pi$

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Abstract

Recently, we pointed out that chiral transformation properties of strong penguin operators change in the transition from unquenched to (partially) quenched QCD. As a consequence, new penguin-like operators appear in the (partially) quenched theory, along with new low-energy constants, which should be interpreted as a quenching artifact. Here, we extend the analysis to the contribution of the new low-energy constants to the $K^0 \to \pi^+\pi^-$ amplitude, at leading order in chiral perturbation theory, and for arbitrary (momentum non-conserving) kinematics. Using these results, we provide a detailed discussion of the intrinsic systematic error due to this (partial) quenching artifact. We also give a simple recipe for the determination of the leading-order low-energy constant parameterizing the new operators in the case of strong LR penguins.

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1 Introduction

A reliable calculation of long-distance contributions to non-leptonic kaon-decay rates, and, in particular, to the CP-violating part parametrized by the quantity ε'/ε has been a long-standing challenge. Ideally, one would expect such calculations to be in the domain of lattice QCD, but in practice many theoretical and numerical difficulties have made progress in this direction rather slow. Recently however, two lattice collaborations have reported on numerical results for both the real and imaginary parts of $\Delta I = 1/2$ and $\Delta I = 3/2$ $K \to \pi\pi$ matrix elements with a rather satisfactory control over statistical errors [1, 2]. These lattice computations were done with the effective weak $\Delta S = 1$ hamiltonian with three flavors, *i.e.* with the charm integrated out, and they were possible because of the use of lattice fermions with good chiral symmetry. Both groups reported values of ε'/ε which are non-zero, and thus consistent with the existence of direct CP violation, but of opposite sign (and comparable size) to the experimentally measured value.

While statistical errors for these lattice computations seem to be reasonably under control, this is not the case for a large class of systematic errors, which will need to be studied further in the future. One source of systematic error is the use of the quenched approximation. In a previous paper [3] we pointed out that, in addition to the fact that quenched QCD is just not the same theory as full QCD, an ambiguity arises in the *definition* of the quenched version of penguin operators appearing in the $\Delta S = 1$ effective weak hamiltonian. The ambiguity originates in the difference of the chiral transformation properties of penguin operators within the quenched and unquenched theories. This implies that not only will quenched lattice results be hampered by the fact that we do not really know whether quenched values of given matrix elements are close to their real-world values, but, in the case of penguins, they also depend on which definition of the operators is chosen, since more than one definition is possible.

In fact, both lattice computations [1, 2] did not directly compute $K \to \pi\pi$ matrix elements, but $K \to \pi$ (with $M_\pi = M_K$) and $K \to 0$ transition amplitudes, and used chiral perturbation theory (ChPT) to convert them into the desired $K \to \pi\pi$ matrix elements [4]. In ref. [3] we explained how the chiral properties of penguin operators change in the transition to the (partially) quenched theory, and how, in principle, more than one definition of a quenched penguin operator is possible. Using ChPT, we traced how this affects $K \to \pi$ and $K \to 0$ matrix elements. We restricted ourselves to LR penguins (*i.e.* Q_5 and Q_6), because the effects in this case already appear at leading order in ChPT, while they are a next-to-leading order effect for LL penguins. Because of the fact that the ambiguity is already present at leading order for matrix elements of $Q_{5,6}$, this may be an important issue for ε'/ε (while it is expected to be less important for the real parts of $K \to \pi\pi$ amplitudes, and thus the $\Delta I = 1/2$ rule).

In this paper we extend our ChPT calculations to the effect of the ambiguity on $K \to \pi\pi$ matrix elements, again to leading chiral order. This is important for two reasons. First, lattice computations may be done directly for $K \to \pi\pi$ matrix elements, and their chiral behavior needs to be known in order to fit lattice results as a function of quark masses.

Since lattice computations are typically done with unphysical (*i.e.* energy/momentum non-conserving) kinematics, we present our results for the most general kinematics possible, both in the quenched and partially quenched cases. Second, once the complete (leading-order) ChPT expressions for $K \to 0$, $K \to \pi$ and $K \to \pi\pi$ matrix elements are available, it is possible to give a more detailed discussion of the systematic error introduced by the ambiguity in the definition of quenched penguin operators.

The paper is organized as follows. In section 2, we review the main observation of ref. [3]. We show how a LR penguin, which transforms in an irreducible representation (irrep) of $SU(3)_L \times SU(3)_R$, splits into two operators in the (partially) quenched theory, with each of them transforming in a different irrep of the (partially) quenched chiral symmetry group. One of these irreps corresponds "naturally" to the single irrep of the unquenched theory, while the other irrep can be considered as "new," and an artifact of quenching. We give the ChPT realization of all relevant operators at leading and next-to-leading chiral order, introducing new low-energy constants (LECs) which appear in correspondence to the new irrep. In section 3, we present our results for $K^0 \to \pi^+\pi^-$ penguin matrix elements to leading order in ChPT, with general kinematics, and specialize these results to physical (*i.e.* energy-momentum conserving) kinematics. In section 4, we discuss different strategies available for using quenched lattice results to estimate real-world $K \to \pi\pi$ matrix elements, and give some numerical examples. We provide a simple prescription for determining the leading-order LEC representing the new irrep in section 5, and section 6 contains our conclusions. Some of this work has already been presented in ref. [5].

2 Review of LR penguins in (partially) quenched QCD and ChPT

A lagrangian definition for partially quenched QCD can be constructed as explained in ref. [6] (see also ref. [7] for an alternative realization using the replica method). In addition to the valence quarks q_{vi}, $i = u, d, s$, with masses m_{vi}, one introduces a separate set of sea quarks q_{si}, $i = 1, \ldots, N$, with masses m_{si}, and a set of "ghost" quarks q_{gi}, $i = u, d, s$, with masses equal to those of the valence quarks $m_{gi} = m_{vi}$ [8]. Ghost quarks are given bosonic statistics, such that the ghost-quark determinant cancels the valence-quark determinant, thus leaving only the sea-quark determinant present in the path integral. Therefore, only sea quarks propagate in internal loops.

Since partially quenched QCD thus contains more flavors than unquenched QCD, its flavor symmetry group is larger than the QCD one. The full chiral symmetry group relevant for light meson physics is the graded extension of the ordinary chiral group $SU(3+N|3)_L \times SU(3+N|3)_R$ [6]. It is graded because part of its elements transform fermions into bosons and *vice versa*. The quenched theory, which has no sea quarks at all, corresponds to the special case $N = 0$ [9].

We consider LR penguin operators of the form

$$Q_{penguin} = (\overline{s}d)_L(\overline{u}u + \overline{d}d + \overline{s}s)_R \ , \tag{2.1}$$

where

$$\begin{aligned} (\overline{q}_1 q_2)_{L,R} &= \overline{q}_1 \gamma_\mu P_{L,R} q_2 \ , \\ P_{L,R} &= \frac{1}{2}(1 \mp \gamma_5) \ , \end{aligned} \tag{2.2}$$

and color contractions are not specified, so that $Q_{penguin}$ can represent both Q_5 and Q_6. As already pointed out in ref. [3], the u, d and s fields in eq. (2.1) represent valence quarks in the partially quenched theory, and the penguin operator can be decomposed as[1]

$$\begin{aligned} Q_{penguin} &= \frac{3}{N}\,\mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\psi\overline{\psi}\gamma_\mu P_R) + \mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(A\psi\overline{\psi}\gamma_\mu P_R)\ , \\ &\equiv \frac{3}{N}\,Q^{PQS}_{penguin} + Q^{PQA}_{penguin}\ , \end{aligned} \tag{2.3}$$

$$A = \mathrm{diag}(1-\frac{3}{N}, 1-\frac{3}{N}, 1-\frac{3}{N}, -\frac{3}{N}, \dots, -\frac{3}{N})\ , \tag{2.4}$$

$$\Lambda_{ij} = \delta_{is}\delta_{jd}\ ,$$

where the first 3 (valence) entries of A are equal to $1-3/N$, and the next $N+3$ (sea and ghost) entries are equal to $-3/N$. Here ψ collects all quark fields in the theory, $\psi = (q_{vi}, q_{si}, q_{gi})$. The tensor Λ projects onto the valence $(\overline{s}d)_L$ term in the first factor of $Q_{penguin}$. The motivation for splitting $Q_{penguin}$ this way is that $Q^{PQS}_{penguin}$ and $Q^{PQA}_{penguin}$ form different representations of the partially quenched symmetry group: $Q^{PQS}_{penguin}$ ($Q^{PQA}_{penguin}$) transforms in the trivial (adjoint) irrep of $SU(3+N|3)_R$. As a consequence, there are at least two different ways of embedding the QCD penguin operator into the partially quenched theory. One is to choose the partially quenched penguin to be a singlet under $SU(3+N|3)_R$, *i.e.* $Q^{PQS}_{penguin}$, as in the unquenched theory, whereas the other choice is to use the original operator, which is seen to be a linear combination of two irreducible operators. This latter choice was made in refs. [1, 2]. In ref. [10] non-singlet penguin operators such as $Q^{PQA}_{penguin}$ were not considered, because singlet factors such as $(\overline{u}u+\overline{d}d+\overline{s}s)_R$ in eq. (2.1) had been implicitly extended to singlets under the full (partially-)quenched symmetry group. Therefore, the analysis of ref. [10] was not complete, and ref. [3] and this paper remedy this for LR penguins.

To leading order (LO), the operators $Q^{PQS}_{penguin}$, $Q^{PQA}_{penguin}$ are represented in ChPT by [3]

$$Q^{PQS}_{penguin} \ \to\ -\alpha_1^{(8,1)}\,\mathrm{str}\,(\Lambda L_\mu L_\mu) + \alpha_2^{(8,1)}\,\mathrm{str}\,(\Lambda X_+)\ , \tag{2.5}$$

$$Q^{PQA}_{penguin} \ \to\ f^2\,\alpha^{(8,8)}\,\mathrm{str}\,(\Lambda\Sigma A\Sigma^\dagger)\ , \tag{2.6}$$

where

$$L_\mu = i\Sigma\partial_\mu\Sigma^\dagger\ , \qquad X_\pm = 2B_0(\Sigma M^\dagger \pm M\Sigma^\dagger)\ , \tag{2.7}$$

[1]In a theory with K valence quarks, all ratios $3/N$ get replaced by K/N.

with M the quark-mass matrix, B_0 the parameter B_0 of ref. [11], $\Sigma = \exp(2i\Phi/f)$ the unitary field describing the partially quenched Goldstone-meson multiplet, and f the bare pion-decay constant normalized such that $f_\pi = 132$ MeV. The α's are the corresponding LECs. Notice that $Q^{PQA}_{penguin}$, unlike $Q^{PQS}_{penguin}$, is of order p^0, due to the fact that the right-handed current in $Q^{PQA}_{penguin}$ is not a partially quenched singlet (*cf.* electro-magnetic penguins[2]). As already observed in ref. [3], the new operator $Q^{PQA}_{penguin}$ does not contribute at tree level to matrix elements with only valence quarks on external lines, since the matrix A is effectively proportional to the unit matrix in the valence sector. Indeed, replacing A by the unit matrix in eq. (2.6) would make the operator vanish. This is no longer true at next-to-leading-order (NLO), *i.e.* at order p^2, where one-loop contributions from $Q^{PQA}_{penguin}$ to valence-quark matrix elements are non-zero.

Since the singlet-operator contributions also start at order p^2, the NLO contributions from $Q^{PQA}_{penguin}$ compete with the LO contributions from $Q^{PQS}_{penguin}$, and thus need to be taken into account already in a leading-order analysis of $K \to 0$, $K \to \pi$ and $K \to \pi\pi$ matrix elements. This also implies that a renormalization scale dependence already appears at leading order for those partially quenched matrix elements. That scale dependence is absorbed by new $O(p^2)$ counterterms for $Q^{PQA}_{penguin}$. The complete list of CPS-even [4] operators is

$$
\begin{aligned}
Q^{PQA}_1 &= \frac{\beta^{(8,8)}_1}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\{\Sigma A\Sigma^\dagger, L_\mu L_\mu\}\right) , \\
Q^{PQA}_2 &= \frac{\beta^{(8,8)}_2}{(4\pi)^2}\,\mathrm{str}\left(\Lambda L_\mu \Sigma A\Sigma^\dagger L_\mu\right) , \\
Q^{PQA}_3 &= \frac{\beta^{(8,8)}_3}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\{\Sigma A\Sigma^\dagger, X_+\}\right) , \\
Q^{PQA}_4 &= \frac{\beta^{(8,8)}_4}{(4\pi)^2}\,\mathrm{str}\left(\Lambda[\Sigma A\Sigma^\dagger, X_-]\right) , \\
Q^{PQA}_5 &= \frac{\beta^{(8,8)}_5}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\Sigma A\Sigma^\dagger\right)\mathrm{str}\left(L_\mu L_\mu\right) , \\
Q^{PQA}_6 &= \frac{\beta^{(8,8)}_6}{(4\pi)^2}\,\mathrm{str}\left(\Lambda\Sigma A\Sigma^\dagger\right)\mathrm{str}\left(X_+\right) , \\
Q^{PQA}_7 &= \frac{\beta^{(8,8)}_7}{(4\pi)^2}\, i\,\partial_\mu\,\mathrm{str}\left(\Lambda[\Sigma A\Sigma^\dagger, L_\mu]\right) ,
\end{aligned}
\tag{2.8}
$$

where we introduced the $O(p^2)$ LECs $\beta^{(8,8)}_{1,\ldots,7}$.

The partially quenched theory with $N = 3$ light sea quarks represents a special case. For $N = 3$, the LECs of the partially quenched theory must be the same as those of the physical, unquenched theory [12], basically because they represent the coefficients in an

[2] In fact, $Q^{PQA}_{penguin}$ is a component of the same irrep as the electro-magnetic penguin, except for $N = 0$ [3].

expansion in powers of quark masses, and thus only depend on the number of dynamical (sea) quarks, and not on the quark masses themselves. Therefore, in the $N=3$ partially quenched theory, what one should do is to omit $Q^{PQA}_{penguin}$ altogether, because the aim is to obtain the values of $\alpha^{(8,1)}_{1,2}$, and not the $(8,8)$ LECs $\alpha^{(8,8)}$ and $\beta^{(8,8)}_i$.[3]

The quenched case, $N=0$, is different. In this case, the decomposition reads

$$
\begin{aligned}
Q^{QCD}_{penguin} &= \frac{1}{2}\,\mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\psi\overline{\psi}\gamma_\mu P_R) + \mathrm{str}\,(\Lambda\psi\overline{\psi}\gamma_\mu P_L)\,\mathrm{str}\,(\hat{N}\psi\overline{\psi}\gamma_\mu P_R)\ , \\
&\equiv \frac{1}{2}\,Q^{QS}_{penguin} + Q^{QNS}_{penguin}\ , \qquad (2.9)\\
\hat{N} &= \frac{1}{2}\mathrm{diag}(1,1,1,-1,-1,-1)\ , \qquad (2.10)
\end{aligned}
$$

where valence entries of $\hat{N}$ are equal to $\frac{1}{2}$, and ghost entries are equal to $-\frac{1}{2}$. The first operator in the decomposition is a singlet under $SU(3|3)_R$, while the second is not (NS for non-singlet). However, $Q^{QNS}_{penguin}$ can transform into the singlet operator, implying that the non-singlet operators do not form a representation by themselves. In other words, $Q^{QNS}_{penguin}$ can mix into $Q^{QS}_{penguin}$, which is possible because $\hat{N}$ is not supertrace-less, unlike A in the partially quenched case. The ChPT realization of both operators is obtained from the expressions given in eqs. (2.6,2.8) and by replacing $A \to \hat{N}$. When referring to the quenched theory, we will add a subscript q to the LECs, and rename[4] the LECs for $Q^{QNS}_{penguin}$ as $\alpha^{(8,8)} \to \alpha^{NS}_q$, $\beta^{(8,8)}_i \to \beta^{NS}_{qi}$.

Within the fully quenched approximation, as also in the partially quenched case with $N \neq 3$, there is no reason that the LECs should have the same values as those of the unquenched theory. In general, the non-analytic terms are modified by quenching, and even the scale dependence of LECs is different between the quenched and unquenched theories. It is thus not *a priori* clear what choice to make for the embedding of penguin operators into the quenched theory. We will return to this issue in section 4 below.

3 Non-singlet $K^0 \to \pi^+\pi^-$ matrix elements with general kinematics

In this section we present the partially quenched and quenched results for the contribution of strong penguin operators to the $K^0 \to \pi^+\pi^-$ matrix element to order p^2 in ChPT. We will restrict ourselves to the isospin limit in the valence sector, $m_{vu} = m_{vd}$, not assuming momentum conservation, so that $q \neq p_1 + p_2$, with q the (ingoing) K^0 momentum and p_1 (p_2) the (outgoing) π^+ (π^-) momentum. All momenta are onshell and we work in euclidean space, *i.e.* $p_1^2 = p_2^2 = -M_\pi^2$, $q^2 = -M_K^2$. M_{jvi} (M_{jsi}) is the mass of a meson made out of the jth valence quark and the ith valence (sea) quark. In all results presented below, we have symmetrized the expressions in the pion momenta p_1 and p_2.

[3]As long as one does not consider electro-magnetic penguin contributions.

[4]In the quenched theory there is no relation between $Q^{QNS}_{penguin}$ and the electro-magnetic penguins [3].

For the contribution of the singlet operator in eq. (2.5) a simple tree-level calculation yields the order p^2 result

$$\begin{aligned}
\langle\pi^+\pi^-|Q^{PQS}|K^0\rangle &= -\frac{4i}{f^3}\left\{\alpha_1^{(8,1)}\left(\frac{1}{2}q(p_1+p_2)+p_1p_2\right)\right. \\
&\left.+\frac{2}{3}\alpha_2^{(8,1)}(M_K^2-M_\pi^2)\left(1+\frac{1}{2}\,\frac{2p_1p_2+q(p_1+p_2)-2M_\pi^2}{(q-p_1-p_2)^2+M_K^2}\right)\right\}\,.
\end{aligned} \tag{3.1}$$

Note that the contribution proportional to $\alpha_2^{(8,1)}$ vanishes for energy-conserving kinematics, because the corresponding operator can be written as a total derivative by using equations of motion [4]. As mentioned before, the tree-level contribution from the new non-singlet operator in eq. (2.6) vanishes. At one loop, it contributes at order p^2. For general kinematics, we obtain, in the isospin limit,

$$\begin{aligned}
\langle\pi^+\pi^-|Q^{PQA}|K^0\rangle_{1-loop} &= \frac{1}{2}[\mathcal{A}(q;p_1,p_2)+\mathcal{A}(q;p_2,p_1)]\,, \\
\mathcal{A}(q;p_1,p_2) &= \frac{4i}{3f^3}\,\alpha^{(8,8)}\sum_{i\ sea}\left\{3(p_1p_2-M_\pi^2)\,I(M_{2si}^2,M_{2si}^2,(p_1+p_2)^2)\right. \\
&+\frac{2M_K^2-M_\pi^2+qp_1}{(q-p_1)^2}\left(L(M_{2si}^2)-L(M_{3si}^2)\right) \\
&-6\,\frac{(qp_1+M_K^2)(qp_1+M_\pi^2)}{(q-p_1)^2}\,I(M_{2si}^2,M_{3si}^2,(q-p_1)^2) \\
&\left.+\frac{(2p_1p_2-2qp_1+4qp_2-2M_\pi^2)}{(q-p_1-p_2)^2+M_K^2}\,(L(M_{3si}^2)-L(M_{2si}^2))\right\} \\
&-\sum_{i\ ghost}(M_{jsi}\to M_{jvi},\ j=2,3)\,,
\end{aligned} \tag{3.2}$$

where we used that $M_{3si}^2-M_{1si}^2=M_K^2-M_\pi^2$ at this order, and

$$\begin{aligned}
L(M^2) &= \int\frac{d^D\ell}{(2\pi)^D}\,\frac{1}{\ell^2+M^2}\,, \\
I(M_1^2,M_2^2,p^2) &= -\int\frac{d^D\ell}{(2\pi)^D}\,\frac{1}{(\ell^2+M_1^2)((\ell-p)^2+M_2^2)}\,,
\end{aligned} \tag{3.3}$$

where D is the number of space-time dimensions. Explicit expressions for these integrals in the $\overline{MS}$ scheme are given in the appendix. The one-loop diagrams contributing to this result are displayed in fig. (1). In particular, the vacuum-tadpole diagram (last diagram in fig. (1)) gives a non-zero contribution for general kinematics which corresponds to the next-to-last line of eq. (3.2). In order to obtain the fully quenched result with no sea quarks at all, one simply drops the sum over sea quarks in eq. (3.2), keeping only the sum over ghost quarks, and replaces $\alpha^{(8,8)}\to\alpha_q^{NS}$. There are no contributions from η' double-poles.

The new $O(p^2)$ counterterms in eq. (2.8) give, again for general kinematics

$$\langle\pi^+\pi^-|Q^{PQA}|K^0\rangle_{ct} = \left(1-\frac{3}{N}\right)\frac{4i}{(4\pi^2)f^3}\left\{\left(2\beta_1^{(8,8)}+\beta_2^{(8,8)}\right)\left(\frac{1}{2}q(p_1+p_2)+p_1p_2\right)\right.$$

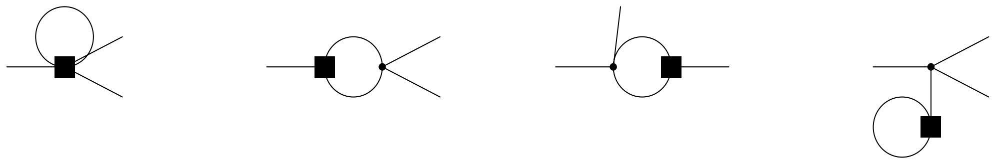

Figure 1: *One-loop diagrams with the insertion of the non-singlet operator at the weak vertex (box) contributing to $K^0 \to \pi^+\pi^-$ with general kinematics*

$$-\frac{4}{3}\beta_3^{(8,8)}(M_K^2 - M_\pi^2)\left(1 + \frac{1}{2}\,\frac{2p_1p_2 + q(p_1+p_2) - 2M_\pi^2}{(q-p_1-p_2)^2 + M_K^2}\right)\Bigg\} \ . \quad (3.4)$$

The structure of these contributions is such that the scale dependence contained in eq. (3.2) can be fully compensated by the LECs $\beta_i^{(8,8)}$, $i = 1, 2, 3$. It is easy to verify that the other LECs $\beta_i^{(8,8)}$, $i = 4, \ldots, 7$ do not contribute. Since external legs only contain valence quarks, the tree-level $K \to \pi\pi$ matrix element does not involve the diagonal A_{ii} elements of the tensor A with i referring to a sea or ghost quark. This implies that, for this calculation, we may replace A with the unit matrix, and the operators Q_i^{PQA} in eq. (2.5) for $i = 4, \ldots, 7$ vanish, while they become proportional to the operators in eq. (2.5) for $i = 1, 2, 3$. It follows that the LECs $\alpha_{1,2}^{(8,1)}$ together with $\beta_{1,2,3}^{(8,8)}$ will always appear in the specific combinations

$$\begin{aligned} &\alpha_1^{(8,1)} - \left(1 - \frac{3}{N}\right)\frac{1}{(4\pi)^2}(2\beta_1^{(8,8)} + \beta_2^{(8,8)}) \ , \\ &\alpha_2^{(8,1)} + \left(1 - \frac{3}{N}\right)\frac{2}{(4\pi)^2}\beta_3^{(8,8)} \end{aligned} \quad (3.5)$$

for all tree-level matrix elements with only valence quarks on the external legs. For the analogue of eq. (3.5) in the quenched case, one replaces the factor $1-3/N$ by $1/2$, $\alpha_i^{(8,1)} \to \alpha_{qi}^{(8,1)}$, $\alpha^{(8,8)} \to \alpha_q^{NS}$ and $\beta_i^{(8,8)} \to \beta_{qi}^{NS}$.

We conclude this section with the expressions for the same matrix elements in the case of "physical" kinematics, *i.e.* with the choice $q = p_1 + p_2$. Setting $q = p_1 + p_2$ and using that $q^2 = -M_K^2$ and $p_1^2 = p_2^2 = -M_\pi^2$, one obtains from eqs. (3.1), (3.2) and (3.4)

$$\langle \pi^+\pi^- | Q^{PQS} | K^0 \rangle^{phys} = \frac{4i}{f^3}\alpha_1^{(8,1)}(M_K^2 - M_\pi^2) \ , \quad (3.6)$$

$$\begin{aligned} \langle \pi^+\pi^- | Q^{PQA} | K^0 \rangle^{phys}_{1-loop} = \ & \frac{4i}{3f^3}\,\alpha^{(8,8)} \sum_{i\ sea}\Bigg\{ -\frac{3}{2}M_K^2\, I(M_{2si}^2, M_{2si}^2, -M_K^2) \\ & - \left(\frac{3}{2}\frac{M_K^4}{M_\pi^2} - 3M_K^2\right)\, I(M_{2si}^2, M_{3si}^2, -M_\pi^2) \\ & - \left(\frac{3}{2}\frac{M_K^2}{M_\pi^2} - 3\right)\,\left[L(M_{2si}^2) - L(M_{3si}^2)\right]\Bigg\} \\ & - \sum_{i\ ghost}(M_{jsi} \to M_{jvi},\ j = 2, 3) \ , \end{aligned} \quad (3.7)$$

$$\langle \pi^+\pi^- | Q^{PQA} | K^0 \rangle^{phys}_{ct} = \frac{-4i}{(4\pi^2)f^3}\left(1 - \frac{3}{N}\right)\left(2\beta_1^{(8,8)} + \beta_2^{(8,8)}\right)(M_K^2 - M_\pi^2) \ . \quad (3.8)$$

Dropping the sea-quark terms and carrying out the sum over ghost quarks, we obtain a more explicit expression for the one-loop contribution in the quenched case, in the $\overline{MS}$ scheme:

$$\begin{aligned}\langle\pi^+\pi^-|Q^{QNS}|K^0\rangle^{phys}_{1-loop} \;=&\; \\ &\frac{i}{16\pi^2 f^3}\alpha_q^{NS}\left\{12(M_K^2-M_\pi^2)\left(\log\frac{M_\pi^2}{\Lambda^2}-1\right)\right. \\ &\qquad+\left(\frac{M_K^6}{M_\pi^4}-2\frac{M_K^4}{M_\pi^2}+2M_K^2\right)\log\frac{M_K^2}{M_\pi^2} \\ &\qquad+\left(\frac{M_K^6}{M_\pi^4}-6\frac{M_K^4}{M_\pi^2}+10M_K^2-4M_\pi^2\right)\log\frac{2M_K^2-M_\pi^2}{M_\pi^2} \\ &+2M_K^2\left(F(M_\pi^2,M_\pi^2,-M_K^2)-2i\,\pi\,\theta(M_K^2-4M_\pi^2)\sqrt{1-\frac{4M_\pi^2}{M_K^2}}+\frac{\pi}{3}\sqrt{3}\right) \\ &\left.+\left(\frac{M_K^4}{M_\pi^2}-2M_K^2\right)\left(2F(M_\pi^2,M_K^2,-M_\pi^2)+F(M_K^2,2M_K^2-M_\pi^2,-M_\pi^2)\right)\right\}\ ,\end{aligned} \tag{3.9}$$

where the function F is given in the appendix.

4 Strategies for quenched estimates of real-world penguin matrix elements

Recent numerical estimates of $K\to\pi\pi$ matrix elements reported in refs. [1, 2] have been obtained via the *indirect* method, where the simpler $K\to\pi$ and $K\to 0$ transition amplitudes are computed on the lattice and then converted into estimates for $K\to\pi\pi$ matrix elements using ChPT. However, the fact that those numerical results are still obtained in the quenched approximation introduces a source of systematic error which is in principle uncontrolled. As already explained, the LECs of the quenched theory do not have to have values equal to those of the unquenched theory. Typically, even the scale dependence of quenched and unquenched LECs is not the same; it depends on the number of light dynamical (sea) quarks in the theory.

In the case of penguin operators, an additional ambiguity arises because it is *a priori* unclear whether it would be best to take matrix elements of $Q^{QCD}_{penguin}$, *i.e.* a linear combination of $Q^{QS}_{penguin}$ and $Q^{QNS}_{penguin}$ as in eq. (2.9), or to drop the contribution from Q^{QNS} under the assumption that $\alpha^{(8,1)}_{q1}$ is the best estimate of $\alpha^{(8,1)}_{1}$.

In order to discuss possible strategies in more detail, we first recall the leading order ChPT expressions for $K\to\pi$ and $K\to 0$ matrix elements of strong penguin operators, from ref. [3]. In the quenched approximation ($N=0$) one has

$$\langle\pi^+|Q^{QCD}_{penguin}|K^+\rangle \;=\; \frac{4M^2}{f^2}\left\{\alpha^{(8,1)}_{q1}-\alpha^{(8,1)}_{q2}-\frac{1}{(4\pi)^2}\left(\beta^{NS}_{q1}+\frac{1}{2}\beta^{NS}_{q2}+\beta^{NS}_{q3}\right)\right\}\ , \tag{4.1}$$

$$\langle 0|Q^{QCD}_{penguin}|K^0\rangle = \frac{4i}{f}\left\{\left(\alpha^{(8,1)}_{q2}+\frac{1}{(4\pi)^2}\beta^{NS}_{q3}\right)(M_K^2-M_\pi^2)\right. \qquad (4.2)$$
$$\left. +\alpha^{NS}_q\sum_{i\ valence}\Big(L(M^2_{3vi})-L(M^2_{2vi})\Big)\right\} ,$$

where $M_K = M_\pi = M$ in the case of the $K\to\pi$ matrix element, and contributions of both singlet and non-singlet operators are included. Notice that only the (quenched versions of the) combinations (3.5) of LECs appear in these expressions, as expected. Assuming that one can limit the analysis to leading-order in ChPT, there are at least three different strategies for estimating $K\to\pi\pi$ penguin matrix elements from LECs obtained by fitting eqs. (4.1,4.2) to quenched numerical results:

1. Ignore α^{NS}_q, but not the other LECs associated with the non-singlet operator, $\beta^{NS}_{q1,2,3}$. Both β^{NS}_{q3} and $\beta^{NS}_{q1}+\frac{1}{2}\beta^{NS}_{q2}$ are scale dependent (however, their sum is not, as can be seen from eq. (4.1)), implying that this strategy is scale dependent. However, it still makes sense in case the non-analytic contribution proportional to α^{NS}_q (*cf.* eq. (3.7)) is numerically small compared to all other contributions at a reasonable scale Λ of order 1 GeV. Thus, the linear combination $\alpha^{(8,1)}_{q1}-\frac{1}{(4\pi)^2}(\beta^{NS}_{q1}+\frac{1}{2}\beta^{NS}_{q2})$ is taken as the best estimate for the unquenched $\alpha^{(8,1)}_1$, and eq. (3.6) can then be used to obtain the physical $K\to\pi\pi$ matrix element (at tree level). This is the strategy followed in refs. [1, 2]. In fact, in these works, it was assumed that the contribution proportional to α^{NS}_q in eq. (4.2) is small.

2. Drop all the non-singlet operators. It was shown in ref. [3] that this can be done by dropping, in the fully quenched case, all *eye*-diagrams in which the right-handed quarks in eq. (2.1) are contracted. This can be easily deduced from eq. (2.9). This strategy was explored for Q_6 in ref. [13]. (For the partially-quenched case, see below.)

3. Perform a complete quenched calculation including all contributions from singlet and non-singlet operators. After extracting all the LECs, singlet and non-singlet, one can use the sum of eqs. (3.6), (3.9) and the quenched version of eq. (3.8) to determine the quenched $K\to\pi\pi$ matrix element at the physical point.

Strategy 2 isolates $\alpha^{(8,1)}_{q1}$, and might thus appear to be the obvious choice, since it is this LEC that is needed for calculating $K\to\pi\pi$ matrix element (to chiral leading-order) in the unquenched theory. However, as we already mentioned, the values of LECs in the quenched and unquenched theories do not have to be equal, and it might happen that (at some scale Λ) the quenched combination $\alpha^{(8,1)}_{q1}-\frac{1}{(4\pi)^2}(\beta^{NS}_{q1}+\frac{1}{2}\beta^{NS}_{q2})$, determined from strategy 1, is indeed a better estimate of $\alpha^{(8,1)}_1$. Strategy 2 can be viewed as the situation in which the strong interactions are quenched at all scales between the weak and hadronic scales, because in that case only singlet penguin operators would appear in the evolution from the weak to the hadronic scale. So, while, on the one hand, it appears natural to assume only a mild flavor dependence of the LECs, in particular $\alpha^{(8,1)}_1$, one might, on the other hand, argue that it is better to calculate the evolution from the weak to the hadronic scale in the unquenched theory, even if the matrix element at the hadronic scale is finally computed in the quenched approximation. The key point is that it is impossible to decide which strategy is best.

The exception to these observations is the case of partially quenched QCD in which the number of light sea quarks is *equal* to that of the real world, in which $N = 3$. In the partially quenched theory, the singlet operator is (*cf.* eq. (2.3))

$$Q^{PQS}_{penguin} = \frac{3}{N}(\overline{s}d)_L \left(\overline{u}_v u_v + \overline{d}_v d_v + \overline{s}_v s_v + \sum_i \overline{q}_{si} q_{si} + \overline{u}_g u_g + \overline{d}_g d_g + \overline{s}_g s_g \right)_R , \tag{4.3}$$

where the subscripts v, s and g denote valence, sea and ghost quarks, respectively. Strategy 2 now corresponds to dropping all diagrams in which the right-handed valence and ghost quarks in the second factor of eq. (4.3) are contracted [3]. If the number of sea quarks $N = 3$ (but with the sea- and valence-quark masses not necessarily equal), the singlet LECs $\alpha^{(8,1)}_{1,2}$ are those of the real world [12], and therefore strategy 2 is the only correct one in this case.

For any other case, fully quenched or partially quenched with $N \neq 3$, there is *a priori* no preferred choice; the spread in results obtained by employing all three strategies should be taken as a (lower bound of the) systematic error due to quenching. The extent to which strategies 1 and 3 lead to numerically different results depends on the size of α^{NS}_q contributions (at a given scale Λ). From eqs. (3.6,3.9) we find, taking physical values for all parameters, $M_K = 500$ MeV, $M_\pi = 140$ MeV, $f = f_\pi = 132$ MeV, $M_\rho = 770$ MeV and $M_\eta = 550$ MeV, that

$$\begin{aligned} -i[K^0 \to \pi^+\pi^-]_q &= 400.7(\alpha^{(8,1)}_{q1} - \alpha^{(27,1)}_q) + (28.2 - 7.2i)\alpha^{NS}_q \quad (\Lambda = 1 \text{ GeV}) , \\ &= 400.7(\alpha^{(8,1)}_{q1} - \alpha^{(27,1)}_q) + (32.2 - 7.2i)\alpha^{NS}_q \quad (\Lambda = M_\rho) , \\ &= 400.7(\alpha^{(8,1)}_{q1} - \alpha^{(27,1)}_q) + (37.3 - 7.2i)\alpha^{NS}_q \quad (\Lambda = M_\eta) , \end{aligned} \tag{4.4}$$

where we added in the tree-level ChPT contribution from the $SU(3)_L$ 27-plet operator [4]. If α^{NS}_q is of the same order as $\alpha^{(8,1)}_{q1} - \alpha^{(27,1)}_q$, the contribution of the terms proportional to α^{NS}_q is indeed small. The smallness of the coefficient of α^{NS}_q is due to a $1/(4\pi)^2$ suppression factor coming from the loop integral, and one might argue that $\alpha^{NS}_q/(4\pi)^2$ is the "natural" parameter to compare with $\alpha^{(8,1)}_{q1} - \alpha^{(27,1)}_q$, in which case the contribution would not be small. Notice also that a small spurious imaginary part is generated by the non-singlet operator via the ghost-pion one-loop rescattering diagram. It is clear that the value of α^{NS}_q will have to be determined from a lattice computation. While this can be done by including the α^{NS}_q terms, of *e.g.* eq. (4.2), in a fit to lattice data, there exists a much simpler and more reliable way of estimating the size of α^{NS}_q, as will be explained in the next section.

Under the assumption that α^{NS}_q can be neglected without introducing a large uncertainty into the final estimate of strong penguin $K \to \pi\pi$ matrix elements, the question remains whether (to leading order in ChPT) $\alpha^{(8,1)}_{q1}$ or $\alpha^{(8,1)}_{q1} - \frac{1}{(4\pi)^2}(\beta^{NS}_{q1} + \frac{1}{2}\beta^{NS}_{q2})$ would be a better estimate of $\alpha_1^{(8,1)}$. The issue was investigated in ref. [13], where it was found that the difference between the two choices is numerically significant. At the physical kaon mass, the numerical value of the B parameter corresponding to Q_6 turns out to be approximately twice as large when the contribution of the non-singlet operator $Q_{penguin}^{QNS}$ is omitted altogether. Translated into estimates for the leading-order LECs, this implies that $\alpha_{q1}^{(8,1)}$ is approximately twice as large as $\alpha_{q1}^{(8,1)} - \frac{1}{(4\pi)^2}(\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS})$. This may lead to substantial modifications in quenched estimates of ε'/ε, as discussed in ref. [5].

We emphasize that the whole discussion here is based on leading-order ChPT, and that NLO contributions may still lead to a substantial correction. However, it is reasonable to believe that NLO effects will not invalidate the basic content of our observations.

Finally, we give a few more numerical examples of the partially quenched case with $N = 2$, always keeping the valence quark masses at their physical values (in the isospin limit), and choosing the two sea quarks to be degenerate in mass. Taking $m_{sea} = m_u = m_d$, we find

$$\begin{aligned} -i[K^0 \to \pi^+\pi^-]_{N=2} &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + 12.7\alpha^{(8,8)} \quad (\Lambda = 1\text{ GeV})\ , \\ &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + 14.0\alpha^{(8,8)} \quad (\Lambda = M_\rho)\ , \\ &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + 15.7\alpha^{(8,8)} \quad (\Lambda = M_\eta)\ , \end{aligned} \tag{4.5}$$

whereas taking $m_{sea} = m_s$ we obtain

$$\begin{aligned} -i[K^0 \to \pi^+\pi^-]_{N=2} &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + (2.8 - 7.2i)\alpha^{(8,8)} \quad (\Lambda = 1\text{ GeV})\ , \\ &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + (4.1 - 7.2i)\alpha^{(8,8)} \quad (\Lambda = M_\rho)\ , \\ &= 400.7(\alpha_1^{(8,1)} - \alpha^{(27,1)}) + (5.8 - 7.2i)\alpha^{(8,8)} \quad (\Lambda = M_\eta)\ . \end{aligned} \tag{4.6}$$

Recall that for the $N = 2$ theory values of the LECs do not have to equal those of the $N = 3$ theory. However, the partially quenched theory with two light sea quarks is closer to the real-world theory than the quenched ($N = 0$) theory. This is reflected by the fact that the coefficients of $\alpha^{(8,8)}$ are small compared to those of α_q^{NS} in eq. (4.4). Notice in addition that in the $N = 2$ case, with $m_{sea} = m_u = m_d$, the small spurious imaginary part vanishes, since it comes entirely from the pion-rescattering loop diagram where the sea-quark contribution is now fully cancelled by the corresponding ghost-quark contribution.

In the case of $N = 3$ sea quarks with masses equal to the three valence quarks, ghost- and sea-quark contributions in eqs. (3.2,3.4) cancel,[5] ¶as they should, because this choice of parameters corresponds precisely to unquenched QCD.

5 How to determine α_q^{NS} on the lattice

In principle, it is possible to determine α_q^{NS} from matrix elements with only physical (valence) particles as external states. For instance, given good enough statistics and a wide

[5]In eq. (3.4) the cancellation already occurs by just setting $N = 3$, because in this tree-level expression the sea- and ghost-quark masses do not appear.

enough range of quark masses, it can be determined from a fit to eq. (4.2). However, as also pointed out in ref. [2], the logarithmic terms in eq. (4.2) can look very linear in the typical range of quark masses used in lattice computations, making it hard to disentangle α_q^{NS} from $\alpha_{q2}^{(8,1)} + \frac{1}{(4\pi)^2}\beta_{q3}^{(8,1)}$. It would therefore be preferable to determine α_q^{NS} from a matrix element to which it contributes at order p^0, because no other operators can "contaminate" the result at that order.

It is very simple to do so, by considering matrix elements with ghost quarks on the external lines instead of valence quarks. Since this corresponds to a flavor rotation on the external lines, one needs to rotate the operator $Q_{penguin}^{QNS}$ accordingly. A key point is that, while of course ghost quarks are not explicitly present in a quenched computation, their propagators are identical to those of the valence quarks, which are available in the actual computation.

So, in order to determine α_q^{NS}, we propose to consider the following matrix element. First, we rotate $Q_{penguin}^{QNS}$ by an $SU(3|3)_L$ rotation into

$$\tilde{Q}_{penguin}^{QNS} = (\overline{s}\gamma_\mu P_L \tilde{d})\,\mathrm{str}\,(\hat{N}\psi\overline{\psi}\gamma_\mu P_R)\ . \tag{5.1}$$

This operator is in the same irrep of the group $SU(3|3)_L \times SU(3|3)_R$, is thus parametrized by the same LECs as $Q_{penguin}^{QNS}$, and in particular, to leading order, by α_q^{NS}. We then consider the matrix element of this operator between a *fermionic* kaon $\tilde{K} \propto \overline{\tilde{d}}\gamma_5 s$ and the vacuum. To leading order,

$$\langle 0|\tilde{Q}_{penguin}^{QNS}|\tilde{K}\rangle = 2if\alpha_q^{NS} + O(p^2)\ , \tag{5.2}$$

thus isolating α_q^{NS}. Carrying out all quark Wick contractions, one finds that

$$\begin{aligned}
\langle 0|\tilde{Q}_{penguin}^{QNS}(y)\overline{\tilde{d}}(x)\gamma_5 s(x)|0\rangle \ &= \\
-\frac{1}{2}\Big\{\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_L\langle\tilde{d}(y)\overline{\tilde{d}}(x)\rangle\right]\mathrm{tr}\Big[\gamma_\mu P_R(&\langle u(y)\overline{u}(y)\rangle + \langle d(y)\overline{d}(y)\rangle + \langle s(y)\overline{s}(y)\rangle \\
&+\langle\tilde{u}(y)\overline{\tilde{u}}(y)\rangle + \langle\tilde{d}(y)\overline{\tilde{d}}(y)\rangle + \langle\tilde{s}(y)\overline{\tilde{s}}(y)\rangle)\Big] \\
&-\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_R\langle s(y)\overline{s}(y)\rangle\gamma_\mu P_L\langle\tilde{d}(y)\overline{\tilde{d}}(x)\rangle\right] \\
&+\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_L\langle\tilde{d}(y)\overline{\tilde{d}}(y)\rangle\gamma_\mu P_R\langle\tilde{d}(y)\overline{\tilde{d}}(x)\rangle\right]\Big\}\ ,
\end{aligned} \tag{5.3}$$

where the traces are over spin and color indices only. A key observation is now that ghost propagators and valence propagators are equal flavor by flavor, $\langle\tilde{d}(y)\overline{\tilde{d}}(x)\rangle = \langle d(y)\overline{d}(x)\rangle$, *etc.*. Using this property, eq. (5.3) simplifies to

$$\begin{aligned}
\langle 0|\tilde{Q}_{penguin}^{QNS}(y)\overline{\tilde{d}}(x)\gamma_5 s(x)|0\rangle \ &= \\
-\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_L\langle d(y)\overline{d}(x)\rangle\right]\,&\mathrm{tr}\left[\gamma_\mu P_R(\langle u(y)\overline{u}(y)\rangle + \langle d(y)\overline{d}(y)\rangle + \langle s(y)\overline{s}(y)\rangle)\right] \\
&+\frac{1}{2}\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_R\langle s(y)\overline{s}(y)\rangle\gamma_\mu P_L\langle d(y)\overline{d}(x)\rangle\right] \\
&-\frac{1}{2}\mathrm{tr}\left[\gamma_5\langle s(x)\overline{s}(y)\rangle\gamma_\mu P_L\langle d(y)\overline{d}(y)\rangle\gamma_\mu P_R\langle d(y)\overline{d}(x)\rangle\right]\ .
\end{aligned} \tag{5.4}$$

We conclude that it is possible to estimate α_q^{NS} as a leading-order effect using only combinations of contractions of valence-quark propagators. For the $K \to 0$ matrix element of $Q_{penguin}^{QNS}$, *cf.* eqs. (2.9,4.2), the contractions in terms of valence quarks are of the same form, but the first two terms have the opposite sign, while the last term has the same sign as in eq. (5.4). Since the $K \to 0$ matrix element is of order p^2, we may combine the two results to obtain

$$\begin{aligned}
-\mathrm{tr}\, & \left[\gamma_5 \langle s(x)\overline{s}(y)\rangle \gamma_\mu P_L \langle d(y)\overline{d}(y)\rangle \gamma_\mu P_R \langle d(y)\overline{d}(x)\rangle\right]_{amputated} \\
&= \sqrt{Z}\left(\langle 0|\tilde{Q}_{penguin}^{QNS}|\tilde{K}\rangle + \langle 0|Q_{penguin}^{QNS}|K\rangle\right) \\
&= \sqrt{Z}\left(2if\alpha_q^{NS} + O(p^2)\right) ,
\end{aligned} \tag{5.5}$$

making it even easier to determine α_q^{NS}. The wave-function renormalization Z is defined from $\overline{d}\gamma_5 s = \sqrt{Z}\, K$.

The analysis for a similar determination of $\alpha^{(8,8)}$ in the partially quenched theory is analogous. There, of course, the observation is not new, since $\alpha^{(8,8)}$ is also the leading LEC for the electro-magnetic penguin (which in the partially quenched theory with $N \geq 1$ is in the same irrep as $Q_{penguin}^{PQA}$ of eq. (2.6) [3]). The main differences between a determination of $\alpha^{(8,8)}$ and α_q^{NS} are that, first, α_q^{NS} is *not* related to the electro-magnetic penguin in the quenched case [3], and second, that in order to determine it using leading-order (in this case $O(p^0)$) ChPT, one is forced to consider ghost quarks, as we did above.

6 Conclusion

In this paper, we continued our investigation of the ambiguities afflicting strong penguin contributions to $K \to \pi\pi$ weak matrix elements due to the use of the quenched approximation.

The fact that the way of *embedding* penguin operators of the effective weak hamiltonian in the quenched theory is not unique tells us that, in the enlarged context of electro-weak interactions, the usual definition of the quenched theory is not complete. If only strong interactions are considered, it is sufficient to define quenched QCD as the modified version of QCD in which the quark determinant is set equal to a constant. A field-theoretic definition can be given through the introduction of ghost quarks into the path-integral [8], giving access to a complete picture of the symmetries of the quenched theory [9]. As soon as one considers operators external to QCD (*i.e.* the addition of electro-weak interactions), one has to answer the question how these operators should be incorporated into the quenched theory. Usually, this is straightforward. One classifies the operator by its flavor quantum numbers, in other words, one determines the irrep of $SU(3)_L \times SU(3)_R$ under which this operator transforms. If there exists a larger irrep of the quenched symmetry group which reduces to the unquenched irrep, the corresponding component of the quenched irrep can be taken as the quenched definition of the operator. However, in the case of strong penguins the operator, while irreducible in the unquenched theory, is a linear combination of components of *more than one* irrep of the quenched symmetry group. Therefore each LEC of the unquenched theory corresponds to a set of LECs in the quenched theory. The ambiguity arises, because there is *a priori* no criterium for which linear combination of quenched LECs (if any) would yield the best estimate of the unquenched LEC. In the case of LR penguins considered here and in ref. [3], this phenomenon produces an effect already at leading order in ChPT.

We remark that even in the simplest case when there exists a one-to-one correspondence between unquenched and quenched irreps, there is still the freedom to choose any component of the quenched irrep, and this flexibility can be used to extract LECs in the most convenient way [14]. However, in this case there is no ambiguity in the relation between unquenched and quenched LECs (even though their values may differ). This is in principle not different from the situation within the unquenched theory, where in general any component of an irrep can be used to extract the corresponding LEC. A classic example for weak matrix elements is the relation between B_K and the $K^+ \to \pi^+\pi^0$ decay rate [15]. (At non-leading order, it may not be possible to determine all LECs describing an operator in ChPT from one process, of course.)

The ambiguity affecting penguin operators is fundamental, since there exists no solid theoretical argument that can be used to decide the issue. Therefore, we argue that one should compare all choices that can be reasonably made, and take the resulting spread of estimated values as a lower bound on the systematic error due to quenching. It appears that in the case of ε'/ε this systematic error is rather large [13, 5]. A leading-order analysis of currently available lattice data [1, 2, 13] seems to indicate that quenched lattice computations cannot even confirm that this parameter is non-vanishing in the Standard Model. It could also be that the large numerical difference found between $\alpha_{q1}^{(8,1)}$ and $\alpha_{q1}^{(8,1)} - \frac{1}{(4\pi)^2}(\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS})$ would be explained by the fact that higher orders in ChPT have not been taken into account, but we consider this to be unlikely. While it is clear that higher orders are numerically important, there appears to be no reason to assume that $\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS}$ is small. It could also be that α_q^{NS}, which appears in $K^0 \to 0$, and needs to be subtracted to obtain $\alpha_{q1}^{(8,1)} - \frac{1}{(4\pi)^2}(\beta_{q1}^{NS} + \frac{1}{2}\beta_{q2}^{NS})$ from $K^+ \to \pi^+$, is not small. This would affect the determination of $\alpha_{q2}^{(8,1)} + \frac{1}{(4\pi)^2}\beta_{q3}^{NS}$, and hence the size of the subtraction. It is therefore important to obtain a reliable estimate of α_q^{NS}. We suggested a simple method for extracting its value.

The above argument does *not* imply that lattice computations of ε'/ε are doomed to fail. On the contrary, quenched estimates of ε'/ε with a particular choice for the strong penguins demonstrate that this computation is feasible, thanks to major advances in both theory and computational power. However, what will be needed in order to eliminate systematic errors due to quenching is a partially quenched study with $N = 3$ light sea quarks. This is the only approximation to unquenched QCD which is reliable in that it can be extrapolated systematically to the real world [12]. Currently existing quenched results give us invaluable information on what is needed to promote them to the required $N = 3$ world. For partially quenched QCD with $N \neq 3$, the situation is essentially the same as for quenched QCD, modulo differences in detail.

Summarizing, we presented the quenched and partially quenched results for the non-singlet contribution to the $K^0 \to \pi^+\pi^-$ matrix element. This made it possible to discuss in detail various strategies one might follow to use quenched computations in order to estimate the real-world value of this amplitude. Since α_q^{NS} contributes to this matrix element, but not to the $K^+ \to \pi^+$ transition amplitude used in ref. [1, 2], this introduces an additional ambiguity already at leading chiral order. The importance of this ambiguity depends on the size of α_q^{NS} and we have proposed a simple recipe for its determination. Our expressions for $K^0 \to \pi^+\pi^-$ with the most general possible kinematics and the inclusion of the non-singlet contributions, are appropriate for the analysis of *direct* quenched computations of this matrix element at leading order in ChPT. Beyond leading order, new problems arise [16], which may invalidate current methods for the *direct* determination of $K \to \pi\pi$ amplitudes with $\Delta I = 1/2$ in quenched and partially quenched QCD.

Appendix

In this appendix we collect explicit expressions for the basic loop integrals appearing in eq. (3.2) *etc.* Using dimensional regularization, we have

$$L(M^2) = \int \frac{d^D\ell}{(2\pi)^D}\,\frac{1}{\ell^2+M^2} = \frac{M^2}{16\pi^2}\left(\left[-\frac{2}{\epsilon}+\gamma-\log 4\pi\right]+\log\frac{M^2}{\Lambda^2}-1\right)\ , \tag{A.1}$$

where Λ is the running scale, $\epsilon = 4-D$, and

$$\begin{aligned}
\mathrm{Re}\,I(M_1^2,M_2^2,p^2) &= -\mathrm{Re}\int \frac{d^D\ell}{(2\pi)^D}\,\frac{1}{(\ell^2+M_1^2)((\ell-p)^2+M_2^2)} \\
&= \frac{1}{16\pi^2}\Bigg\{\left[-\frac{2}{\epsilon}+\gamma-\log 4\pi\right] \\
&\quad -1+\log\frac{M_2^2}{\Lambda^2}+\frac{1}{2}\left(1-\frac{M_1^2}{p^2}+\frac{M_2^2}{p^2}\right)\log\frac{M_1^2}{M_2^2}+\frac{1}{2}F(M_1^2,M_2^2,p^2)\Bigg\}\ ,
\end{aligned} \tag{A.2}$$

in which

$$\begin{aligned}
F(M_1^2,M_2^2,p^2) &= \sqrt{\lambda\left(1,\frac{M_1^2}{p^2},\frac{M_2^2}{p^2}\right)}\,\log\frac{p^2+M_1^2+M_2^2+p^2\sqrt{\lambda(1,M_1^2/p^2,M_2^2/p^2)}}{p^2+M_1^2+M_2^2-p^2\sqrt{\lambda(1,M_1^2/p^2,M_2^2/p^2)}}\ , \\
\lambda(x,y,z) &= (x-y+z)^2+4xy\ .
\end{aligned} \tag{A.3}$$

$\overline{MS}$ expressions are obtained by dropping the contact terms in square brackets.

For $p^2 > 0$, the argument of the logarithm in eq. (A.3) is positive, and I is real. For $p^2 \le 0$, F is obtained by analytic continuation. $\lambda(1, M_1^2/p^2, M_2^2/p^2)$ turns negative for $-(M_1+M_2)^2 < p^2 < -(M_1-M_2)^2$, and we find that I is still real with F now given by

$$F(M_1^2,M_2^2,p^2) = 2\sqrt{-\lambda\left(1,\frac{M_1^2}{p^2},\frac{M_2^2}{p^2}\right)}\,\arctan\frac{-p^2\sqrt{-\lambda(1,M_1^2/p^2,M_2^2/p^2)}}{p^2+M_1^2+M_2^2}\ . \tag{A.4}$$

At $p^2 = -(M_1^2 + M_2^2)$ the argument of the arctangent has a singularity, across which the branch of the arctangent has to be chosen continuously:

$$\arctan \frac{-p^2\sqrt{-\lambda(1, M_1^2/p^2, M_2^2/p^2)}}{p^2 + M_1^2 + M_2^2} \quad = \quad \mathrm{Arctan}\, \frac{-p^2\sqrt{-\lambda(1, M_1^2/p^2, M_2^2/p^2)}}{p^2 + M_1^2 + M_2^2} + \pi \ , \qquad p^2 < -(M_1^2 + M_2^2) \ , \tag{A.5}$$

where Arctan denotes the principal value of the arctangent. Again continuing analytically across $p^2 = -(M_1 + M_2)^2$, F is again given by eq. (A.3), but $I(M_1^2, M_2^2, p^2)$ picks up an imaginary part:

$$\mathrm{Im}\, I(M_1^2, M_2^2, p^2) = -\frac{1}{16\pi^2}\, \pi \sqrt{\lambda\left(1, \frac{M_1^2}{p^2}, \frac{M_2^2}{p^2}\right)} \ . \tag{A.6}$$

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